MECHANICS, CLASSICAL

At the most general level, 'classical mechanics' covers the approach to physical phenomena that dominated science from roughly the time of Galileo until the early decades of this century. The approach is usually characterized by the assumption that bodies carry an inherent mass and well defined positions and velocities. Bodies subsist within a three dimensional absolute space and influence one another through forces that work reciprocally to one another. These objects obey the three laws of motion articulated by Isaac Newton in 1686 in a deterministic manner: once a mechanical system is assembled, its future behavior is rigidly fixed. Such 'classical' assumptions were eventually rejected by Einstein's theory of relativity, where the assumption of a three dimensional Euclidean space is abandoned, and by quantum mechanics, where determinism and well defined positions and velocity are eschewed.

Classical mechanics is frequently characterized as 'billiard ball mechanics' or 'the theory of mechanism' on the grounds that the science treats its materials in the manner of colliding particles or clockwork. The reader should approach such stereotypes with caution because the basic framework of classical mechanics has long been subject to divergent interpretations that unpack the content of Newton’s "three laws" in remarkably different ways. These differing interpretations provide incompatable catalogs of the basic objects that are supposed to comprise the 'classical world'--should they be point masses, rigid bodies or truly flexible substances? Or, as many writers have suggested, should mechanics not be regarded as 'about' the world at all, but merely as a source of useful but fictitious idealizations of reality?

These foundational disagreements explain why classical mechanics has often found itself entangled in metaphysics. The list of prominent philosophers whose thought was centrally concerned with the constitution of 'classical' matter is lengthy: Hobbes, Descartes, Berkeley, Leibniz, Kant, Whitehead,... To these can be added the many scientists who have contributed importantly to philosophy: Newton, Euler, Cauchy, Helmholtz, Hertz, Poincare, Mach, Kirchhoff,... Much of modern philosophy of science is characterized by attitudes that were originally articulated during the nineteenth century's attempts to clarify the grounds of classical mechanics.

1-3 NEWTON'S LAWS OF MOTION
4 POINT MASSES AND IDEALIZATION
5 RIGID BODIES AND CONSTRAINTS
6 CONTINUA
7 RETROSPECT
1 NEWTON'S LAWS OF MOTION

Introductory textbooks usually sketch a simple, standardized approach to the
content of classical mechanics, which, for reasons discussed in Sect. 4, may be called the point mass interpretation. Although this interpretation is often presented as 'the correct understanding of classical mechanics', few philosophers or physicists of the classical era would have accepted this reading as adequate. The point mass reading not only suffers empirical difficulties, it does not describe a universe that would have been pleasing to most practitioners. The point mass reading has gained its textbook centrality largely because alternative foundations for mechanics prove resistant to easy articulation. Indeed, satisfactory alternatives were not fully developed until well into this century (largely by the engineering community which requires a trustworthy classical basis for its endeavors). The mismatch between the conceptual simplicity of textbook orthodoxy and the expected characteristics of the subject is apt to cause students of philosophy much difficulty, largely because the basic tensions at the heart of classical mechanics are rarely laid out explicitly.

The essential content of classical doctrine is commonly held to reside in Newton's celebrated laws of motion. Unfortunately, these principles, in the form they were articulated by Newton, are subject to a surprising range of interpretations. A good entry into the conceptual difficulties of classical mechanics begins by considering these laws and the basic readings to which they have been subjected.

It should be mentioned that the topic of electromagnetism is largely left out of account here. This is partially because 'classical' electromagnetism, as the subject is understood today, fits more neatly into a relativistic universe than a Newtonian framework. In the nineteenth century, light and other electromagnetic activities were regarded as vibrations within an 'aether' that was conceived as a material continuum in the manner of Sect. 6. See FIELDS, CLASSICAL THEORY OF.

In Thompson and Tait's translation, Newton's laws are:

**FIRST LAW**: Every body continues in its state of rest or of uniform motion in a straight line, except insofar as it may be compelled by force to change that state.

**SECOND LAW**: Change of motion is proportional to force applied, and takes place in the direction of the straight line in which the force acts.

**THIRD LAW**: To every action there is always an equal and contrary reaction: or, the mutual actions of any two bodies are always equal and oppositely directed.

Newton borrowed the first law, also known as 'the principle of inertia', from
Galileo and Descartes (although neither would have accepted the doctrine in its fullest generality). The law claims that the natural motion of a particle uninfluenced by external forces is always a straight line, whereas earlier scientists had often presumed such natural notions to be circular or directed towards a 'natural home' in the universe. 'Inertia' is the traditional term for the tendency of a body to retain its present state of motion; it is harder to stop a cannon ball than a baseball even if they travel at the same velocity. From a Newtonian point of view, 'inertia' is simply another name for 'mass', but it is common to speak of 'inertial forces' as well, a notion discussed in Sect. 3.

Henri Poincare claimed that the first law (as well as much of the rest of the Newtonian framework) does not report genuine physical content, but instead represents a mere 'convention'. Newton presumed that an Euclidean space exists independently of matter and understood his first law as delineating how free matter will move upon this geometrical stage. Poincare objected that we can have no knowledge of spatial structure independent of the behavior of material objects found within it. The first law is not a straightforward 'fact', but merely articulates a criteria for erecting a geometry around material objects. No empirical facts can force a rejection of Newton's first law 'criterion' for the presence of 'straight lines', although, were the empirical facts otherwise, an alternative 'convention' might be favored where the notions of geometry become coordinated to the movements of matter in a different fashion. This conventionalism has proved enormously influential within twentieth century analytic philosophy and much controversy still hovers over whether Poincare's implied distinction between a 'convention' and a 'genuine fact of nature' can be made out.

Newton has been widely criticized for assuming the existence of an enduring 'absolute space' in the first law, because (i) the notion sounds crazily 'metaphysical'; (ii) its presence seems inherently unobservable; or (iii) the third law of motion (on its strongest reading) forces a principle of Galilean relativity: no law of nature allow a determination of whether a set of objects lies at rest in absolute space or merely moves through it at a constant velocity (reference objects that move in either of these ways define inertial frames). One is therefore denied any means of relocating a point in absolute space after an interval of time passes.

The first two worries are discussed in SPACE AND TIME ???. During the nineteenth century, followers of Kant saw the problem of Galilean relativity as evidence that absolute space does not represent a real condition found in nature, but merely reflects an organizational structure imposed upon reality by the human mind (Poincare's conventionalism shares many affinities with such neo-Kantian thinking). But more sophisticated techniques articulated in this century (chiefly by
E. Cartan) have suggested another resolution to the relativity problem: classical mechanics should follow Einstein's lead in rejecting the notion of an enduring space in favor of a blend of space and time. In Cartan's reformulation, the first law becomes interpreted as demanding a certain geometrical structure within the melded spacetime, avoiding the unobservable aspects of Newton's persisting space. Although this approach weakens Newton's geometrical assumptions somewhat, it agrees that spacetime possesses objective features independently of human decision or the placement of material objects within the spacetime.

Cartan also showed that gravitation could be 'geometrized away' within certain classical frameworks, in imitation of the techniques of general relativity. It should be mentioned, however, that Galilean relativity was not universally accepted during the heyday of classical mechanics--various electrical effects once seemed as if they might provide criteria for detecting inertial movement within absolute space.

2 Newton's second law is popularly known as 'F = ma', where 'F' stands for 'force', 'm' for 'mass' and 'a' for 'acceleration'. The claim is that a particle will deviate from its natural straight line motion only insofar as a force is imposed from without, in which case the particle will accelerate away from that straight line to a degree conditioned by its mass. In a modern treatment (if uncomplicated by 'constraints'), 'F = ma' provides the skeleton upon which the basic equations of motion for a collection of particles are constructed. The recipe is this. (a) Delineate the particles whose interactions one wishes to study; (b) Determine what kinds of specific forces act between these (do they gravitationally attract one another? do they display electrical attraction or repulsion?, etc.); (c) For each particle \( \mathbf{a} \), decompose each of forces that act upon \( \mathbf{a} \) along a selected Cartesian axis--for example, \( F_x \) might express the amount of electrical force that acts upon particle \( \mathbf{a} \) along the x-axis; (d) For each particle \( \mathbf{a} \) and each Cartesian direction \( x \), assemble all of the specific forces under (c) into a sum called the 'total force acting upon the particle acting in direction \( x \)'; (e) Set this sum of forces equal to \( m \frac{d^2 x}{dt^2} \) (i.e., to what in the calculus is called 'the second derivative of \( \mathbf{a} \)'s location along the x-axis'). The upshot is what the mathematician calls 'a system of \( 3n \) second-order ordinary differential equations' for a collection of \( n \) particles. The list under (b) of specific force laws working within the collection are called the constitutive principles for the system at hand; crudely speaking, they delineate the basic manner in which forces bind the collection together. The equations of motion constructed by this recipe then fix (modulo some occasional technicalities) how the system will behave at all future times once the positions and velocities of all
particles are known at some specific time (such data represent the system's 'phase' or initial condition). That is, the equations of motion in classical physics determine the future states of the system evolve from its initial condition (usually past states are determined as well). The determinism of classical physics is a famous bugbear of ethical philosophy--see *FREE WILL*. Quantum mechanics no longer accepts determinism of the classical variety, but it is dubious that what it substitutes is preferable from an ethical point of view.

'Determinism' is not the same as 'predictability' in any reasonable sense. It has been realized for some time that the deterministic behavior of most classical systems is so complicated that no computer could accurately possibly track their behavior for more than a few seconds. Much of the recent fascination with so-called 'chaos' has evolved from the discovery of new ways to extract useful partial information from these otherwise unpredictable systems. Motivated largely by worries in thermodynamics, some researchers have proposed removing the 'determinism' from classical mechanics by harnessing the 'functional analysis' techniques used in quantum theory. Such ideas remain sketchy at present.

The second law recipe has been described in some detail because Newton's less explicit formulation has provided a ripe occasion for philosophical confusion. Our 'recipe' approach becomes clearly established only within the investigations of Leonhard Euler in the eighteenth century. Expressing the principle as simply 'F = ma' encourages the impression that the second law is not really an axiom, but merely a definition of 'force'. This objection overlooks the non-arbitrary decomposition into specific forces required in the recipe. Some writers have complained that Newton's laws are 'empty' because they describe systems either subject to one force (the second law) or no forces at all (the first law). But no real life system is like this. In the author's evaluation, this objection, and many others like it, fails to understand the Euler recipe correctly.

But if Newton's second law is understood in this vein, physics should be greatly concerned to flesh out the complete set of specific force laws used in step (b). Newton's law of universal gravitation supplies an admirable example of the kind of law needed, but, beyond that paradigm, classical physicists were somewhat lackadaisical in their efforts to fill out the missing laws (they never agreed, for example, on the nature of the forces that glue matter together, for example). Joseph Sneed nonetheless sees the demand for these as yet unarticulated force laws as a vital part of the content of classical physics--he hopes that Thomas Kuhn's celebrated notion of a 'scientific paradigm' can be clarified in this manner. In truth, the constraint tradition, described in Sect. 5, often herded mechanics' actual development in directions other than the hypothesized search for specific force
laws. As we shall see, the criticism of 'F = ma' as an empty definition gains a greater cogency under the point of view of this school.

In general, many of the standard objections to Newton's laws originally grew out of substantial disputes over the fundamental trustworthiness of key mechanical propositions. As these origins were forgotten, the disputes became reinterpreted (particularly by the logical positivist school) as rather trivial squabbles over whether forces are 'observable' or not. Or so this author thinks.

3 The haziness of Newton's third law--'action = reaction', as it is popularly known--is palpable in its very formulation. The textbooks often add three additional tenets to the claim. (a) All forces arise between pairs of particles. (b) Such forces are directed along the line between them (the forces are 'central'). (c) The strength of these forces depends only upon the spatial separation between the bodies and not, say, upon their relative velocities. With these props in place, one can demand that if \( a \) exerts a specific force \( f \) upon \( b \), then \( b \) will exert a reciprocating force upon \( a \) equal in magnitude to \( f \) but reversed in direction. Although Newton's own law of gravitation suits these requirements, it is doubtful that he would have accepted the (a-c) supplements. Requirement (c) stands in apparent conflict with most frictional forces because their strength depends upon the rate that bodies slip past one another. (b) seems incompatible with sheering forces, as arise when one layer of water slips over another. Each layer exerts a retarding force upon the other, but these forces point along the layers' interface, not 'the line between them'.

The main motivation for including requirements (a-c) is that, on this approach, they are needed to prove fundamental tenets such as the conservation of energy. For example, (c) justifies the notion of 'potential energy'. Newton was unconcerned with these requirements because he had no notion that energy conservation holds. Indeed, he often understood the third law in a variant 'inertial force' manner which will now be explained.

Consider the sun and its planets. On the (a-c) interpretation, 'action = reaction' requires that each planet should exert a gravitational force upon the sun equal in strength to the solar gravitational tug that it feels. Since the sun is more massive than the planets, these planetary attractions will not accelerate it greatly. It does not lead to great mathematical error if the sun is treated as if it is the source of an unreciprocated force--e.g., the solar attractions are described as dependent only upon the planet's position within an inertial frame. On the (a-c) reading, this treatment is only a useful fiction--the reciprocating pulls of the planets must dislodge the sun slightly from its assumed inertial motion. Disregarding the sun's
minor wobbings allows the problem of planetary movements to be effaced from
the complications of solar motion. When third law requirements can be
overlooked in this manner, the force is called external; otherwise, they are dubbed
internal.

However, a long tradition exists of regarding some external forces as non-
approximate in origin. How does a mechanical system balance such a force?
Answer: it develops an 'inertial force of resistance'--its parts accelerate in such a
manner that the sum over all particles of mass times acceleration exactly matches
the external force. Most modern texts dismiss this approach to the third law as a
mistake; no force should be regarded as truly external and inertial reactions do not
qualify as forces at all. Indeed, the 'inertial force' reading seems merely to repeat
the content of the second law over again.

The 'inertial force' understanding of the third law becomes more natural
within the constraint based approach of Sect. 5, where central forces drop from
view. In the mechanics of continua (Sect. 6), the 'principle of material frame
indifference' reveals some further affinities between inertial reactions and forces.

4 POINT MASSES AND IDEALIZATION

Our discussion has involved much hazy talk of 'particles'; what kind of
object is intended by this term? On the interpretation presented so far, a 'particle'
needs to be a point mass: a movable point carrying a fixed mass (and possibly
other quantities such as charge). Forces have been required to 'act along the line
between particles'; this makes obvious sense only if 'particles' lie at uniquely
defined points. Point masses must always remain discrete from one another, for
the possibility of fusing would permit an infinite source of potential energy. On a
point mass interpretation, all forces are of action at a distance type, for they
originate in one particle, yet act, across an intervening vacuum, somewhere else.
Contact forces, where one body exerts a pressure immediately upon another
through geometrical contact, are not tolerated on this approach to Newton's laws.

The point mass approach, standard in most modern textbooks, was first
proposed by R.J. Boscovitch in 1758. Boscovitch's suggestion was rarely
embraced during the heyday of classical mechanics. Most practitioners felt that
their subject should deal directly with some variety of extended body, and not
regard them merely as swarms of points. Newton, although he was often
interesting in arguing that objects in apparent contact were actually separated (vide
his discussion of 'Newton's rings'), also believed that atoms possessed hard cores of
a certain size.

The point mass presentation of classical physics has gained great favor in
modern textbooks partially because of a natural association between its mathematical structures and those of elementary quantum mechanics. 'Correspondence rules' exist to convert point mass equations to the partial differential equations of quantum theory. Under this transition, the quantum particles gain a surrogate for 'size' due to considerations peculiar to the newer theory, e.g., the exclusion principle. Many authors have been mislead by such mathematical relationships into presuming that remarkable successes of classical physics in treating the large scale phenomena of ordinary life can be easily explained by the way in which point mass theory 'reduces' to quantum theory. This impression is misleading, because the point mass interpretation at best provides an awkward basis for setting up the successful models of fluids and elastic bodies that are more adequately established following the program of Sect. 6.

Sometimes it is clearly useful to treat extended bodies as if they were point particles. The locus classicus of this technique is Newton's observation that a rigid, perfectly spherical planet behaves gravitationally from its exterior as if it were a single point carrying the same mass. The orbit of a planet can then be calculated without worrying about its interior. Real planets, of course, are neither perfect globes nor rigid and the predictions of point mass celestial mechanics are slightly inaccurate as a result. Nonetheless, this approximate effacement of the gross behavior of the planets from their internal complications has inspired a popular 'as if' reading of Newton's laws: they delineate a range of idealized models that hopefully suit real life situations to a tolerable degree. When greater accuracy is desired, one shifts to modeling the planet as a large swarm of point masses. Each finer scale of idealized modeling provides predictions of increased accuracy, but we should never pretend that our constructions supply anything other than simplified portraits of reality.

On this approach, it becomes difficult to see how Newton's laws could be shown false; at best, they might prove an inconvenient series of approximations to utilize. An important distinction should be drawn here: in any scientific theory, the successful treatment of everyday phenomena will require artificial simplification because the mathematics pertinent to a fully realistic approach is horrendously complex. But the 'idealization' thesis maintains that, quite apart from mathematical intractability, science must study simplified models due to an inherent lack of reality within the very fabric of physics. However, the 'lack of reality' in point masses may merely demonstrate that they do not provide the best foundation for a satisfactory classical mechanics.

5 CONSTRAINTS AND RIGID BODIES
A universe of point masses seems a far cry from a standard stereotypes of classical mechanics: isn't the subject supposed to treat the universe as a 'great clock'? Aren't clocks composed of gears, rods and springs--extended bodies all? There is a variant tradition, stretching back to antiquity, that exploits the properties of rigid bodies in a direct, geometrical way (gears and rods, but not springs, are rigid bodies). Typically, such members interact with another through contact along their boundaries: two gears intermesh; a bead slides along a wire; a wheel rolls along a plane. These linkages are specified by so-called constraints: equations that geometrically link the movements of distinct bodies. The presence of constraints clearly modifies Newtonian behavior in various ways. For example, our bead, if no external forces act against it, will slide along the wire at a constant velocity forever (if the wire itself is not accelerated). Constraints alter Newton's law of inertia from 'travels with constant rectilinear velocity' to 'travels at an unchanging speed along the curvilinear constraint of the wire'. Principles of unhampered motion that are adapted to accommodate constraints are called generalized laws of inertia.

The existence of constraints, strictly speaking, is incompatible with the point mass reading. Let the bead slide past point $p$ upon the wire. What force will the wire near $p$ exert upon the bead as it slides past? 'Easy', the stock reply goes, 'compute the acceleration $a$ needed to move the bead past $p$. Divide $a$ by the mass of the bead to obtain the force exerted by the wire'. But let the bead slide past $p$ with a greater velocity. The wire must generate a greater force to hold the bead in its rightful place at $p$. In short, the wire must be able to 'see' the bead's velocity before the force needed to maintain the constraint can be exerted. But the point mass understanding of the third law requires that forces be velocity independent; otherwise, conservation of energy might fail.

What should be done? Either constraints are regarded as convenient approximations to point mass reality or our fundamental principles must be modified to tolerate them. In most texts, constraints are introduced in so cavalier a fashion that the policy followed cannot be determined. On the first strategy, 'rigid bodies' are simply collections of particles welded together by very strong (but finite) forces. The masses within our wire must shift slightly when a faster bead passes, although these movements may be quite microscopic. In 1788, however, J.L. Lagrange articulated a version of mechanics that incorporates genuine constraints and rigid bodies within its foundations. He based his approach upon the law of virtual work (sometimes: virtual velocity) and d'Alembert's principle. The idea behind virtual work is illustrated nicely by the behavior of a lever. Work can accomplished if weights are allowed to drop--falling water drives old
fashioned grist mills. String a series of masses along a lever arm; the rigidity of the arm links these weights together by a constraint. How must these weights be arranged if the lever is to stay in equilibrium (= at rest)? Answer: any work that might be gained if the weights on one side of the arm are allowed to fall must be expended to raise the weights strung along the other side. 'Virtual work' declares that equilibrium within any constrained set of mechanical parts will obtain just in case their hypothetical abilities to perform work cancel out in the manner described. Laws of this variety (which predate Lagrange) are called variational principles because a system's actual behavior (equilibrium, in this case) is adduced from a comparison of hypothetical behaviors (i.e., imaginary movements given to the weights). For systems not at rest, Lagrange appealed to d'Alembert's principle. This doctrine (named in honor of an important eighteenth century scientist) claims that if unbalanced external forces are applied to a mechanical system, it will display inertial reactions that exactly balance the external set. Recall from Sect. 3 that an 'inertial reaction' is simply the acceleration of a body multiplied by its mass. D'Alembert's principle thus maintains that the system 'reacts' to external 'activity' by moving against the activity. We noted that Newton's third law is sometimes interpreted in this manner.

Classical mechanics is often presented using other forms of variational technique, e.g., Hamilton's principle and other varieties of 'least action'. Although elementary textbooks often allege that such variants are equivalent to one another and to Newton's original set of laws, this is not true unless a number of background assumptions are made. Hamilton's treatment, especially, renders salient some important features of energy conserving systems, considered with respect to the so-called 'Hamiltonian flow within phase space'. But these matters must be left to standard texts.

Notice that, in dealing with the force upon the bead, we calculated in a pattern reverse to what we might have expected. In celestial calculations, we compute the acceleration of the earth from a prior knowledge of the forces that apply to it. Newton's law of gravitation is the specific force law employed here. But in the bead case, we did not work with any specific force law; instead, the force was calculated merely from the acceleration that the wire requires of the bead. Forces induced by the satisfaction of constraints are traditionally called 'forces of reaction'.

Lagrange's approach tolerates both 'forces of reaction' and law specified forces, where the latter are of action at a distance type. It had long been the hope of mechanics, in its 'clockwork' inspired moods, that action at distance effects might turn out to be the result of contact movements within a hidden medium
linking the distant bodies. Descartes and Leibniz believed that the attraction of planets resulted from swimming in a common 'sea' that pushed them together. Appeal to action at a distance forces was considered a retreat to the 'occult' qualities of medieval times. In the nineteenth century, J.C. Maxwell suggested that electromagnetic effects might be communicated between distant bodies through a complicated set of intervening mechanical parts. Such proposals led Hertz to reformulate mechanics based upon constraints alone (a wider range of these is needed than appear in standard Lagrangian practice). Hertz replaces Newton's laws by a simple principle of 'generalized inertia' and identifies all forms of energy with actual energy of movement (kinetic energy). Most importantly, the notion of force is essentially discarded in Hertz' system, since only the derivative 'forces of reaction' are tolerated in his framework. Hertz' system provides the best realization of the philosophical dream of a truly machine-inspired mechanics. Interest in Hertz' project faded with the rise of quantum theory, but its distrust of the Newtonian notion of 'force' has proved longer lived. In the author's opinion, 'force' should occasion no unhappiness as long as mechanics remains faithful to a point mass interpretation; it is the frequent appeal to post hoc 'forces of reaction' that generates the uneasy sense that 'force' functions as an unnecessary ingredient within mechanics.

In any case, the notion that the universe could actually move as one great machine is less plausible than one might expect. Machines are designed (insofar as possible) so that their parts slip over each other without binding. If a collection of rigid parts is randomly assembled, without the guidance of a designer, they are likely to end in overconstraint: the expected conditions of sliding, rolling, etc. among its parts cannot be satisfied all at once. To accommodate the overconstraint, some of the parts will need to distort or fracture, thus losing their postulated rigidity.

In other words, once the basic 'particles' of mechanics are granted any spatial extension at all, they are likely to lose their postulated rigidity at some time or another. How does this eventuality affect Lagrange's program for the foundations of mechanics? A retreat to an 'idealization' defense is possible: mechanics tries to idealize real physical systems in terms of constrained rigid bodies, despite their lack of permanent reality. Some advantage over point mass idealization is gained, for a better set of modeling tools have been assembled. It would be nice, however, if an approach to mechanics could be found that accepts bodies which are capable of distortion at all levels of analysis. The materials in this reformulated mechanics should be truly flexible entities such as springs and fluids, which can smoothly distort or flow when the right external pressures are exerted. Such objects are
6. CONTINUA

If one tries to describe continua directly, one quickly discovers that formulating plausible laws for genuinely flexible bodies introduces many conceptual problems. To begin, all of the problems of the 'infinitely divided' that have plagued philosophy and mathematics since the time of Zeno's paradoxes arise. Mechanics supplements these with some peculiar difficulties of its own.

For example, it seems natural to interpret the term 'particle' in Newton's laws as an 'infinitesimal portion of a continuous body'. If such a 'particle' belongs to a macroscopic body that stretches, must not that 'particle' enlarge infinitesimally under the stretching as well? But how can a point \( p \) sensibly expand to become a 'bigger point'? And what should \( F = ma \) require of \( p \)? If \( p \) enlarges under an applied force, should its 'mass' (or, more properly, its density) also alter? Traditional arguments concerning 'infinitesimals' were plagued by vagueness of this sort, as critics such as Bishop Berkeley swiftly observed. In the author's opinion, Leibniz's celebrated philosophy of 'monads' represented, in part, a sophisticated attempt to surmount some of these problems by moving outside of the arena of space and time altogether.

Although infinitesimal readings of Newton's second law are common in textbooks, the considered modern opinion holds that this is a mistake: \( F = ma \) should apply only to the extended parts of a continuous body. This shift in perspective introduces many unexpected conceptual subtleties. Suppose that \( A \) is a continuous body with external forces applied at various spots along its outer edge and within its interior. If \( A \) is a perfectly rigid substance, all of these forces can be added together as if they act at a common location. In a rigid body, a force applied at \( p \) can be shifted to any other position \( q \) and still pull \( A \) equally to the right. Applying a force at \( p \) rather than \( q \) affects only how \( A \) will rotate, not how it translates. Indeed, by defining the so-called 'moment' of \( A \)'s array of forces (roughly, a measure of their 'off-centerness'), Euler showed how \( A \)'s rotation could be calculated (the principle employed is called 'balance of angular momentum' or 'moment of momentum'). Euler's two principles--non-infinitesimal \( F = ma \) and 'moment of momentum'--completely settle how any rigid body will respond to applied forces.

But the success of these principles appear to trade upon \( A \)'s postulated rigidity. If \( A \) is composed of flexible stuff, then how it reacts to a schedule of applied forces will clearly reflect the materials of which it is made--the same set of forces will move rubber in a completely different fashion than water. To make
further progress, we must consider the contact forces that arise upon the surfaces inside A that separate points p and q. Draw an imaginary boundary shell B around q that excludes p. An external force applied at p can transmit an indirect influence to q if it successively alters the conditions of the shells of matter that surround q. As the matter outside of B changes its state, a new set of forces will be applied against the surface of B by direct contact. These 'contact forces' will work their way successively through the shells surrounding q and eventually affect q itself. The mathematical characteristics of such 'contact forces' are rather different than those of the action-at-a-distance types considered previously. In the 1820's, A. Cauchy showed how these two types of forces could be harmonized in a fashion that supplies an adequate foundation for continuum mechanics. He first assumed that 'F = ma' holds for (almost) every shell B enclosing q. 'F' represents the resultant of summing the contact forces around B, supplemented by whatever external forces operate within B's interior. 'F = ma' then assigns an overall 'acceleration' to B. Unfortunately, these observations still allow individual points inside B to move in any of the numerous variety of ways that are consistent with the averaged acceleration that non-infinitesimal 'F= ma' assigns to B. But a framework of laws is not adequate to continua until the principles that govern point motion within flexible body are articulated. At this juncture, Cauchy had the brilliant insight to realize that, as the volume of B shrinks to 0, Euler's two principles insure that a rather abstract assemblage of numbers, called the stress tensor, needs to be defined at each point q. Cauchy's 'stress' turns out to be the novel ingredient that renders the mechanics of continua quite distinct from the other formulations of classical physics we have considered. Popular books often encourage the impression that 'stress' is merely a synonym for 'force', but Cauchy's tensor actually supplies an abstract replacement for the naive idea that a 'point' like q might possess an 'infinitesimal surface' upon which the surrounding materials can tug and shear in different directions. In other words, 'stress' provides a subtle means of avoiding the paradoxical 'points with surfaces' that conceptually bedeviled earlier attempts to discuss continua.

With Cauchy's new notion in hand, the differences between water and rubber can be traced to the distinct manners in which states of stress affect stretching within the materials. The law of stretching for rubber--its so-called 'constitutive equation'--looks quite different from the law suitable for water. Such 'constitutive principles' together with Euler's two laws form the core of doctrine upon which continuum mechanics rests.

Cauchy's direct approach to continua usually provides a more reliable guide to the behavior of real life materials than if they are modeled as arrays of classical
points or rigid bodies. Some of the nineteenth century hostility to 'molecules' stemmed from a recognition of this empirical superiority (the historical dispute over the so-called 'rariconstant' theory of elasticity was a case in point). But Cauchy's program is complicated, and, without sufficient care, one can quickly return to conceptual absurdities such as 'points with infinitesimal surfaces'. Many modern texts treat continua as large swarms of point masses simply because this approach provides a 'quick and dirty' way of getting some key classical models up and running, without needing to worry about delicate foundational considerations. Handy appeals to the necessity of 'idealization' camouflage some otherwise awkward non sequiturs. But if one hopes to appreciate the conceptual issues with which a Leibniz would have struggled, the difficulties of continua need to be addressed more honestly.

In point of historical record, a variant program for approaching continua based upon 'strain energy' rather than 'stress' was pursued in the late Nineteenth Century by the so-called 'energetic school'. See HARMAN for further details.

7. RETROSPECT

Which of these approaches to mechanics should the scientists of the nineteenth century have favored? On the one hand, they had little doubt that the apparent continua of real life--strings, fluids, etc.--are lumpish in composition, possibly suggesting the suitability of rigid bodies as the basic objects of physics. But such molecules also seemed to be capable of vibration, indicating that the molecular 'lumps' might possibly act as flexible bodies at a microscopic level of analysis (the 'aether' that was thought to surround the molecules also acted like a continuum as well). Such uncertainties spawned lively arguments over the proper basis of classical mechanics, disputes that were never resolved simply because Nature decided to turn quantum mechanical at just the point where experiments might have settled matters.

Unfortunately, many popular accounts of the history of science and philosophy often forget these genuine conceptual worries and presume that 'classical mechanics' is correctly interpreted only from a point mass point of view. The many philosophers and scientists who struggled to articulate variant conceptions are frequently dismissed as incompetents blind to the true mechanical light. But the deep philosophical heritage of classical mechanics is better appreciated if its basic internal tensions are more sympathetically understood.

References and Further Reading


Friedman, Michael (1972) Foundations of Space-Time Theories, Princeton. (Account of Cartan's work by a philosopher).


* Hertz, Heinrich (1900) The Principles of Mechanics, New York, Macmillan. (Discussed in Sect. 4; preface is wonderful).


* Newton, Isaac (1962), Principia, Berkeley, University of California. (The classic, but hard to read).

* Poincare, Henri (1952) Science and Hypothesis, New York, Dover. (Parts II-III out the 'conventionalism' discussed in Sect. 1).

* Sneed, Joseph (1979) The Logical Structure of Mathematical Physics, Dordrecht, Reidel. (Mentioned in Sect. 2)

* Thompson, William and P. G. Tait (1879) A Treatise on Natural Philosophy, Cambridge. (A classic of nineteenth century thought; quoted in Section 1).


------------- (1977) A First Course in Rational Continuum Mechanics, New York, Academic. (The many works by Truesdell supply the best guide to the continuum tradition of Sect. 6. The 'First Course' is advanced).